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Computation of $H \to gg$ in FDH and DRED: renormalization, operator mixing, and explicit two-loop results

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The $H \to gg$ amplitude relevant for Higgs production via gluon fusion is computed in the four-dimensional helicity scheme (FDH) and in dimensional reduction (DRED) at the two-loop level. The required renormalization is developed and described in detail, including the treatment of evanescent ϵ-scalar contributions. In FDH and DRED there are additional dimension-5 operators generating the Hgg vertices, where g can either be a gluon or an ϵ-scalar. An appropriate operator basis is given and the operator mixing through renormalization is described. The results of the present paper provide building blocks for further computations, and they allow to complete the study of the infrared divergence structure of two-loop amplitudes in FDH and DRED.

Contents

1 Introduction

Higgs production via gluon fusion is one of the most important LHC processes. Its computation at higher orders requires renormalization and factorization to cancel UV and IR divergences. The renormalization is less trivial than the one of standard QCD processes due to the required renormalization of non-renormalizable operators. The virtual corrections have been computed in conventional dimensional regularization (CDR) [1–5]; the required theory of operator renormalization in CDR has been developed in Ref. [6], based on general work in Refs. [7, 8].

In the past years, several alternative regularization schemes have been developed. Purely four-dimensional schemes such as implicit regularization [9, 10] and FDR [11] have

been proposed and used to compute processes of practical interest such as $H \to \gamma\gamma$ [12, 13] and $H \to gg$ [14]. The present paper is devoted to regularization by dimensional reduction (DRED) [15] and the related four-dimensional helicity (FDH) scheme [16]. Both schemes are actually the same regarding UV renormalization, but they differ in the treatment of external partons related to IR divergences.[1] There has been significant progress in the understanding of FDH and DRED: the equivalence to CDR [20, 21], mathematical consistency and the quantum action principle [22], infrared factorization [23, 24] have been established — these results solved several problems that had been reported earlier, related to violation of unitarity [25], Siegel's inconsistency [26], and the factorization problem of [27, 28]. In addition, explicit multi-loop calculations have been carried out [29–33].

More recently, the multi-loop IR divergence structure of FDH and DRED amplitudes has been studied in Ref. [34]. It has been shown that IR divergences in FDH and DRED can be described by a generalization of the CDR formulas given in Refs. [35–38]. The description involves IR anomalous dimensions γ^i for each parton type i. In Ref. [34] they have been computed for the cases of quarks and gluons by comparing the general IR factorization formulas with explicit results for the quark and gluon form factor. In FDH and DRED, however, the gluon can be decomposed into a D-dimensional gluon $\hat{g}$ and $(4-D)$ additional degrees of freedom, so-called ϵ-scalars $\tilde{g}$. In DRED, ϵ-scalars also appear as external states.

The present paper is devoted to a detailed two-loop computation of the amplitude $H \to gg$ in FDH and DRED. In DRED, this involves the computations of $H \to \hat{g}\hat{g}$ and $H \to \tilde{g}\tilde{g}$, since the external gluons can either be gauge fields or ϵ-scalars. The FDH result is identical to the one for $H \to \hat{g}\hat{g}$ and has already been given in Ref. [34], but we will provide further details here.

This detailed computation is of interest for two reasons: First, it provides the basis for obtaining the remaining IR anomalous dimension for ϵ-scalars at the two-loop level. Second, it provides an example of the required renormalization in FDH and DRED, including operator renormalization and operator mixing. The difficulty of renormalization in FDH and DRED, particularly in connection with $H \to gg$, has been pointed out e. g. in Refs. [33, 39].

The outline of the paper is as follows: Section 2 gives a brief description of the regularization schemes and of the relevant Lagrangian and operators. It ends with a detailed list of the required ingredients of the calculation. Apart from the actual two-loop computation and ordinary parameter and field renormalization that are described in Sections 3 and 4, respectively, the main difficulty lies in the renormalization and mixing of the operators generating $H \to gg$. This is discussed in general in Section 5, and specific two-loop results are presented in Section 6. Section 7 then provides the final results for the on-shell amplitudes for $H \to \hat{g}\hat{g}$ and $H \to \tilde{g}\tilde{g}$. The appendix contains details on our projection operators and gives Feynman rules for the different operator insertions.

[1] Parts of the literature, e. g. Refs. [17–19] used the term DR/dimensional reduction for what is called FDH here.

	CDR	HV	FDH	DRED
internal gluon	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
external gluon	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$

Table 1. Treatment of internal and external gluons in the four different regularization schemes, i.e. prescription which metric tensor has to be used in propagator numerators and polarization sums.

2 Regularization schemes and $H \to gg$

It is useful to distinguish the following regularization schemes [24]: conventional dimensional regularization (CDR), the 't Hooft-Veltman (HV) scheme, the four-dimensional helicity (FDH) scheme, and dimensional reduction (DRED). In all these schemes, momenta are treated in $D = 4-2\epsilon$ dimensions (the associated space is denoted by QDS with metric tensor $\hat{g}^{\mu\nu}$). In order to define the schemes, one also needs an additional quasi-4-dimensional space ($Q4S$, metric $g^{\mu\nu}$) and the original 4-dimensional space ($4S$, metric $\bar{g}^{\mu\nu}$). The treatment of gluons in the four schemes is given in Tab. 1. In the table, "internal" gluons are defined as either virtual gluons that are part of a one-particle irreducible loop diagram or, for real correction diagrams, gluons in the initial or final state that are collinear or soft. "External gluons" are defined as all other gluons.

Mathematical consistency and D-dimensional gauge invariance require that $Q4S \supset QDS \supset 4S$ and forbid to identify $g^{\mu\nu}$ and $\bar{g}^{\mu\nu}$. Details can be found in Refs. [22, 24, 34]. The most important relations for the present paper are

$$g^{\mu\nu} = \hat{g}^{\mu\nu} + \tilde{g}^{\mu\nu}, \quad \hat{g}^{\mu\rho}\tilde{g}_{\rho}{}^{\nu} = 0, \quad \hat{g}^{\mu\rho}\bar{g}_{\rho}{}^{\nu} = \bar{g}^{\mu\nu}, \quad \hat{g}^{\mu\nu}\hat{g}_{\mu\nu} = D, \quad \tilde{g}^{\mu\nu}\tilde{g}_{\mu\nu} = N_\epsilon, \tag{2.1}$$

where a complementary 2ϵ-dimensional metric tensor $\tilde{g}^{\mu\nu}$ has been introduced. With these metric tensors we can decompose a quasi-4-dimensional gluon field A^μ as

$$A^\mu = \hat{g}^{\mu\nu}A_\nu + \tilde{g}^{\mu\nu}A_\nu = \hat{A}^\mu + \tilde{A}^\mu \tag{2.2}$$

into a D-dimensional gauge field $\hat{A}^\mu$ and an associated ϵ-scalar field $\tilde{A}^\mu$ with multiplicity $N_\epsilon = 2\epsilon$.[2] Correspondingly, there are two types of particles in the regularized theory: D-dimensional gluons $\hat{g}$ and ϵ-scalars $\tilde{g}$. The unregularized external gluons $\bar{g}$ of FDH are a part of $\hat{g}$.

The regularized Lagrangian of massless QCD then reads

$$\mathcal{L}_{QCD,\,\text{regularized}} = -\frac{1}{4}\hat{F}_a^{\mu\nu}\hat{F}_{\mu\nu,a} - \frac{1}{2\xi}(\partial^\mu \hat{A}_{\mu,a})^2 + i\,\overline{\psi}\hat{\not{D}}\psi + \partial^\mu\overline{c}_a\hat{D}_\mu c_a + \mathcal{L}_\epsilon, \tag{2.3a}$$

$$\mathcal{L}_\epsilon = -\frac{1}{2}(\hat{D}^\mu\tilde{A}^\nu)_a(\hat{D}_\mu\tilde{A}_\nu)_a - g_e\,\overline{\psi}\tilde{\not{A}}\psi - \frac{1}{4!}\left(g_{4\epsilon}^2\right)^{\alpha\beta\gamma\delta}_{abcd}\,\tilde{A}_{\alpha,a}\tilde{A}_{\beta,b}\tilde{A}_{\gamma,c}\tilde{A}_{\delta,d}. \tag{2.3b}$$

[2] In many applications of FDH the dimensionality of $Q4S$ is left as a variable D_s, which is eventually set to $D_s = 4$. The multiplicity of ϵ-scalars is then $N_\epsilon = D_s - D$.

Here, $\hat{F}^{\mu\nu}$ and $\hat{D}^{\mu} = \partial^{\mu} + ig_s\hat{A}^{\mu}$ denote the non-abelian field strength tensor and the covariant derivative in D dimensions; ψ and c are the quark and ghost fields. In Eq. (2.3b) the coupling of ϵ-scalars to (anti-)quarks is given by the evanescent Yukawa-like coupling g_e. This could in principle be set equal to the strong coupling g_s. But, since both couplings renormalize differently this would only hold at tree-level and for one particular renormalization scale [20]; the same is true for the quartic ϵ-scalar coupling $g_{4\epsilon}$. In Eq. (2.3b) we introduce an abbreviation that includes the appearing Lorentz and color structure: $(g_{4\epsilon}^2)^{\alpha\beta\gamma\delta}_{abcd} := g_{4\epsilon}^2(f_{abe}f_{cde}\tilde{g}^{\alpha\gamma}\tilde{g}^{\beta\delta} + \text{perm.})$, where "perm." denotes the 5 permutations arising from symmetrization in the multi-indices $(a,\alpha)\dots(c,\gamma)$. In the following we use all couplings in the form $\alpha_i = \frac{g_i^2}{4\pi}$ with $i = s, e, 4\epsilon$.

The process $H \to gg$ is generated by an effective Lagrangian which arises from integrating out the top quark in the Standard Model. In CDR it contains only the term $-\frac{1}{4}\lambda H \hat{F}_a^{\mu\nu}\hat{F}_{\mu\nu,a}$. In FDH and DRED one again has to distinguish several gauge invariant structures containing either D-dimensional gluons or ϵ-scalars. The effective Lagrangian can be written as

$$\mathcal{L}_{\text{eff}} = \lambda H O_1 + \lambda_\epsilon H \tilde{O}_1 + \sum_i \lambda_{4\epsilon,i} H \tilde{O}_{4\epsilon,i}, \tag{2.4}$$

with

$$O_1 = -\frac{1}{4}\hat{F}_a^{\mu\nu}\hat{F}_{\mu\nu,a}, \tag{2.5a}$$

$$\tilde{O}_1 = -\frac{1}{2}(\hat{D}^{\mu}\tilde{A}^{\nu})_a(\hat{D}_{\mu}\tilde{A}_{\nu})_a. \tag{2.5b}$$

$\tilde{O}_{4\epsilon,i}$ denote operators involving products of four ϵ-scalars. Such operators are not important in the present paper and will not be given explicitly. Like for α_s, α_e and $\alpha_{4\epsilon}$, the couplings λ and λ_ϵ can be set equal at tree-level, but they renormalize differently and have different β functions.

Our final goal is the calculation of the two-loop form factors for gluons and ϵ-scalars. This requires the on-shell calculation of the 3-point function $\Gamma_{HA^{\mu}A^{\nu}}(q,-p,-r)$. All momenta are defined as incoming, so $q = p + r$. The 3-point function can be separated into $\Gamma_{H\hat{A}^{\mu}\hat{A}^{\nu}}$ and $\Gamma_{H\tilde{A}^{\mu}\tilde{A}^{\nu}}$, corresponding to the amplitudes for $H \to \hat{g}\hat{g}$ and $H \to \tilde{g}\tilde{g}$, respectively. In DRED, both on-shell amplitudes are needed according to Tab. 1. In FDH, only $H \to \bar{g}\bar{g}$ is needed, which however is identical to $H \to \hat{g}\hat{g}$ and will not be discussed seperately.

The on-shell calculation requires the knowledge of the two-loop renormalization constants $\delta Z_{\lambda}^{2\text{L}}$ and $\delta Z_{\lambda_\epsilon}^{2\text{L}}$. These in turn can be obtained from an off-shell calculation of $\Gamma_{HA^{\mu}A^{\nu}}$. Projectors extracting the required renormalization constants from the off-shell Green functions and precisely defining the gluon and ϵ-scalar form factors are given in appendix A.1.

We have now all ingredients to discuss the classes of Feynman diagrams that contribute to $\Gamma_{HA^{\mu}A^{\nu}}$ in FDH and DRED:

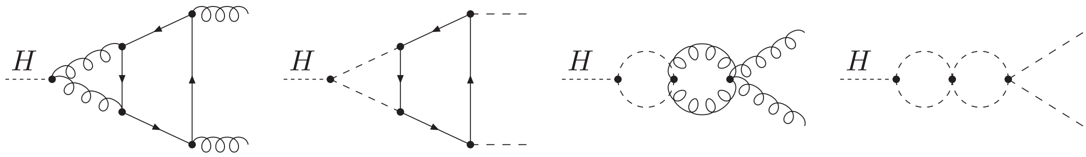

Figure 1. Sample two-loop diagrams for the process $H \to \hat{g}\hat{g}$ and $H \to \tilde{g}\tilde{g}$ in DRED. ϵ-scalars are denoted by dashed lines. The appearing coupling combinations from left to right are $\lambda\alpha_s^2$, $\lambda_\epsilon\alpha_e^2$, $\lambda_\epsilon\alpha_s^2$, $\lambda_\epsilon\alpha_{4\epsilon}^2$.

1. Genuine two-loop diagrams $\Gamma^{\text{2L}}_{HA^\mu A^\nu}$. Some remarks concerning the calculation are presented in Sec. 3.

2. Counterterm diagrams $\Gamma^{\text{1LCT,a}}_{HA^\mu A^\nu}$ and $\Gamma^{\text{2LCT,a}}_{HA^\mu A^\nu}$ arising from one- and two-loop renormalization of the fields, the gauge parameter ξ, and of the couplings α_s, α_e, and $\alpha_{4\epsilon}$. The required renormalization constants are presented in Sec. 4.

3. Counterterm diagrams $\Gamma^{\text{1LCT,b}}_{HA^\mu A^\nu}$ arising from one-loop renormalization of the effective Lagrangian (2.4) at the one-loop level, which includes the renormalization of λ and λ_ϵ. This is a major complication and will be presented in Sec. 5.

4. Overall two-loop counterterm diagrams $\Gamma^{\text{2LCT,b}}_{HA^\mu A^\nu}$ arising from the two-loop renormalization of the effective Lagrangian (2.4), equivalently from the renormalization constants $\delta Z^{\text{2L}}_\lambda$ and $\delta Z^{\text{2L}}_{\lambda_\epsilon}$. These renormalization constants are generally defined by the requirement that the appropriate off-shell Green functions are UV finite after renormalization. For the case of δZ_λ, an elegant alternative determination is possible [6], but that method fails for $\delta Z_{\lambda_\epsilon}$. The results for $\delta Z^{\text{2L}}_\lambda$ and $\delta Z^{\text{2L}}_{\lambda_\epsilon}$ are presented in Sec. 6.

3 Genuine two-loop diagrams

As mentioned above the Green function $\Gamma_{HA^\mu A^\nu}$ can be separated into $\Gamma_{H\hat{A}^\mu\hat{A}^\nu}$ and $\Gamma_{H\tilde{A}^\mu\tilde{A}^\nu}$, corresponding to $H \to \hat{g}\hat{g}$ and $H \to \tilde{g}\tilde{g}$. Examples for genuine two-loop diagrams with either external gluons or ϵ-scalars are shown in Fig. 1.

All loop calculations have been performed using the following setup: the generation of diagrams and analytical expressions is done with the Mathematica package FeynArts [40]; to cope with the extended Lorentz structure in $Q4S$ we use a modified version of TRACER [41]; all planar on-shell integrals are reduced and evaluated with an inplementation of an in-house algorithm that is based on integration-by-parts methods and the Laporta-algorithm [42]; all non-planar and off-shell integrals are reduced and evaluated with the packages FIRE [43] and FIESTA [44].

4 Parameter and field renormalization in FDH and DRED

We now consider the counterterm contributions $\Gamma^{\text{1LCT,a}}_{HA^\mu A^\nu}$ and $\Gamma^{\text{2LCT,a}}_{HA^\mu A^\nu}$. They are given by diagrams exemplified in Fig. 2, where the counterterm insertions are generated by the usual

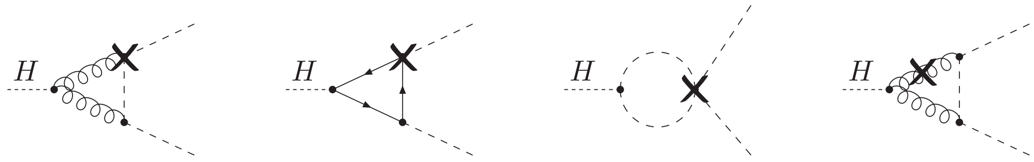

Figure 2. Sample one-loop counterterm diagrams originating from the renormalization of the couplings α_s, α_e, $\alpha_{4\epsilon}$, and of the gauge parameter ξ, respectively.

multiplicative QCD renormalization of the couplings and fields present in Eq. (2.3b). In the following we present the values of the required β functions and anomalous dimensions, which govern the renormalization constants.

4.1 β functions

The renormalization of the couplings α_s, α_e, and $\alpha_{4\epsilon}$ is done by replacing the bare couplings with the renormalized ones. As renormalization scheme we choose a modified version of the $\overline{\text{MS}}$ scheme: like in Ref. [34] we treat the multiplicity N_ϵ of the ϵ-scalars as an initially arbitrary quantity and subtract divergences of the form $\left(\frac{N_\epsilon}{\epsilon}\right)^n$. As a consequence, the corresponding β functions depend on N_ϵ: $\overline{\beta}^i \equiv \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\left(\frac{\alpha_i}{4\pi}\right) = \overline{\beta}^i(\alpha_s, \alpha_e, \alpha_{4\epsilon}, N_\epsilon)$, with $i = s, e, 4\epsilon$. They are given in Refs. [33, 34] and read:

$$
\begin{aligned}
\overline{\beta}^s = &-\left(\frac{\alpha_s}{4\pi}\right)^2 \left[C_A\left(\frac{11}{3} - \frac{N_\epsilon}{6}\right) - \frac{2}{3}N_F\right] \\
&-\left(\frac{\alpha_s}{4\pi}\right)^3 \left[C_A^2\left(\frac{34}{3} - \frac{7}{3}N_\epsilon\right) - \frac{10}{3}C_A N_F - 2C_F N_F\right] \\
&-\left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{\alpha_e}{4\pi}\right) \left[C_F N_F N_\epsilon\right] + \mathcal{O}(\alpha^4),
\end{aligned}
\tag{4.1a}
$$

$$
\overline{\beta}^e = -\left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) 6\,C_F - \left(\frac{\alpha_e}{4\pi}\right)^2 \left[C_A(2 - N_\epsilon) + C_F(-4 + N_\epsilon) - N_F\right] + \mathcal{O}(\alpha^3). \tag{4.1b}
$$

The renormalization of the quartic coupling $(\alpha_{4\epsilon})^{\alpha\beta\gamma\delta}_{abcd}$ is more complicated since the tree-level color structure, $f_{abe}f_{cde}$, is not preserved under renormalization [20]. In the case of an SU(3) gauge group one therefore has to introduce three quartic couplings, $\alpha_{4\epsilon,i}$ with $i = 1, 2, 3$, each of them related to one specific color structure in a basis of color space. Examples for such a basis are given e. g. in Refs. [29, 30].

In the present case of $H \to gg$ the renormalization constant for $\alpha_{4\epsilon}$ only appears in

diagrams like the third of Fig. 2. Hence, only the following contracted β function is needed:

$$
\begin{aligned}
(\overline{\beta}^{4\epsilon})^{\alpha\beta\gamma\delta}_{abcd}\,\delta_{ab}\,\tilde{g}_{\alpha\beta} = \Bigg\{ & \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^2 (9+6\,N_\epsilon) \;\; + \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_{4\epsilon}}{4\pi}\right) C_A^2\,(1-N_\epsilon)\,12 \\
& + \left(\frac{\alpha_e}{4\pi}\right)^2 \Big[C_A N_F (4-2\,N_\epsilon) + C_F N_F (-8-4\,N_\epsilon)\Big] \\
& + \left(\frac{\alpha_e}{4\pi}\right)\left(\frac{\alpha_{4\epsilon}}{4\pi}\right) C_A N_F\,(1-N_\epsilon)(-4) \\
& + \left(\frac{\alpha_{4\epsilon}}{4\pi}\right)^2 C_A^2\,(1-N_\epsilon)(-7-2\,N_\epsilon)\Bigg\}\,\delta_{cd}\,\tilde{g}^{\gamma\delta} + \mathcal{O}(\alpha^3).
\end{aligned}
\tag{4.2}
$$

This result is obtained from a direct off-shell calculation. It agrees with a general result from [45].

4.2 Anomalous dimensions

For the off-shell calculation of $\Gamma_{HA^\mu A^\nu}$ also renormalization of the fields and of the gauge parameter ξ is needed. The renormalization of ξ is fixed by the requirement that the gauge fixing term does not renormalize: $\xi \to Z_{\hat{A}}\xi$. The anomalous dimensions $\gamma_i = \mu^2 \frac{d}{d\mu^2} \ln Z_i$ of gluon and ϵ-scalar fields are obtained from a direct off-shell calculation of the respective two-loop self energies. Their values up to two-loop level read:

$$
\begin{aligned}
\gamma_{\hat{A}} = & -\left(\frac{\alpha_s}{4\pi}\right)\left[C_A\left(\frac{13}{6}-\frac{\xi}{2}-\frac{N_\epsilon}{6}\right)-\frac{2}{3}N_F\right] \\
& -\left(\frac{\alpha_s}{4\pi}\right)^2\left[C_A^2\left(\frac{59}{8}-\frac{11}{8}\xi-\frac{\xi^2}{4}-\frac{15}{8}N_\epsilon\right)-\frac{5}{2}C_A N_F - 2\,C_F N_F\right] \\
& -\left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) C_F N_F N_\epsilon + \mathcal{O}(\alpha^3),
\end{aligned}
\tag{4.3a}
$$

$$
\begin{aligned}
\gamma_{\tilde{A}} = & -\left(\frac{\alpha_s}{4\pi}\right) C_A (3-\xi) - \left(\frac{\alpha_e}{4\pi}\right)\Big[-N_F\Big] \\
& -\left(\frac{\alpha_s}{4\pi}\right)^2\left[C_A^2\left(\frac{61}{6}-2\xi-\frac{\xi^2}{4}-\frac{11}{12}N_\epsilon\right)-\frac{5}{3}C_A N_F\right] - \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right)\Big[-5\,C_F N_F\Big] \\
& -\left(\frac{\alpha_e}{4\pi}\right)^2\left[C_A N_F\left(-1+\frac{N_\epsilon}{2}\right)+C_F N_F\left(2+\frac{N_\epsilon}{2}\right)\right] - \left(\frac{\alpha_{4\epsilon}}{4\pi}\right)^2 C_A^2\,(1-N_\epsilon)\,\frac{3}{4} \\
& + \mathcal{O}(\alpha^3).
\end{aligned}
\tag{4.3b}
$$

Setting N_ϵ and α_e to zero in Eq. (4.3a) yields the well-known gluon anomalous dimension in CDR, see e. g. [46]. The value of $\gamma_{\tilde{A}}$ agrees with the general result for the anomalous dimension of a scalar field [45], confirming the point of view that ϵ-scalars behave like ordinary scalar fields with multiplicity N_ϵ.

5 Operator renormalization and mixing in FDH and DRED

The second type of counterterm contributions, denoted by $\Gamma^{\rm 1lCT,b}_{HA^\mu A^\nu}$ and $\Gamma^{\rm 2lCT,b}_{HA^\mu A^\nu}$, originates from the necessary renormalization of the effective Lagrangian (2.4), equivalently of the operators O_1 and $\tilde{O}_1$. One major difficulty is that multiplicative renormalization of the parameters λ and λ_ϵ is not sufficient since the operators mix with further operators. We will show that the full operator mixing involving gauge non-invariant operators has to be taken into account. The renormalization constants cannot be predicted from known QCD renormalization constants but need to be determined from an off-shell calculation. The general theory of operator mixing in gauge theories and the classification of gauge invariant and gauge non-invariant operators has been developed long ago [7, 8, 47].

In the following we briefly describe operator mixing in the much simpler case of CDR and then explain the cases of FDH and DRED, which involve further operators.

5.1 Operators in CDR

In CDR, a useful basis of scalar dimension-4 operators, which is closed under renormalization, is given in Ref. [6]:

$$O_1 = -\frac{1}{4}\hat{F}^{\mu\nu}_a \hat{F}_{\mu\nu,a}, \tag{5.1a}$$

$$O_2 = 0, \tag{5.1b}$$

$$O_3 = \frac{i}{2}\,\overline{\psi}\,\overleftrightarrow{\hat{\not{D}}}\,\psi, \tag{5.1c}$$

$$O_4 = \hat{A}^\nu_a(\hat{D}^\mu \hat{F}_{\mu\nu})_a - g_s\overline{\psi}\hat{\not{A}}\psi - (\partial^\mu\overline{c}_a)(\partial_\mu c_a), \tag{5.1d}$$

$$O_5 = (D^\mu\partial_\mu\overline{c})_a c_a. \tag{5.1e}$$

Operator O_1 is gauge invariant and related to coupling renormalization; $O_2 = m\overline{\psi}\psi$ in Ref. [6] and corresponds to the fermion mass renormalization; we set $m = 0$. All other operators are constrained by BRS invariance and Slavnov-Taylor identities [7, 8]; operators O_4 and O_5 are not gauge invariant. The basis is chosen such that O_3, O_4 and O_5 are related to field renormalization of ψ, $\hat{A}^\mu$ and c, respectively. In particular, the first two terms of O_4 are generated by applying the functional derivative

$$\hat{A}^\nu_a(x)\frac{\delta}{\delta\hat{A}^\nu_a(x)} \tag{5.2}$$

on the gauge invariant part of the QCD action; the remaining term is then required by BRS invariance and the non-renormalization of the gauge fixing term.[3]

[3] See Refs. [8, 47] for more details; the full operator O_4 can be obtained from evaluating $WY_{\hat{A}^\nu_a}\hat{A}^\nu_a + W(\partial^\nu\overline{c}_a)A_{\nu,a}$, where W is the linearized Slavnov-Taylor operator and $Y_{\hat{A}^\nu_a}$ is the source of the BRS transformation of $\hat{A}^\nu_a$ in the functional integral. Since W is nilpotent, this definition shows that O_4 is compatible with BRS invariance and the Slavnov-Taylor identity and can appear in the operator mixing.

The operators renormalize as

$$O_i \to Z_{ij} O_{j,\text{bare}}, \tag{5.3}$$

where $O_{j,\text{bare}}$ arises from O_j by replacing all parameters and fields by the respective bare quantities. Following an elegant proof in Ref. [6] the nontrivial CDR renormalization matrix Z_{ij} can be written in the form

$$Z_{ij} = \delta_{ij} + \mathbb{D}_i \ln \mathbb{Z}_j. \tag{5.4}$$

Here, $\mathbb{D}_i$ are derivatives with respect to parameters and $\mathbb{Z}_j$ are combinations of ordinary QCD renormalization constants. As a result, in particular the renormalization of Z_{11} is given by

$$Z_{11} = 1 + \alpha_s \frac{\partial}{\partial \alpha_s} \ln Z_{\alpha_s}, \tag{5.5}$$

with the multiplicative renormalization constant of α_s, Z_{α_s}. In this way the renormalization of the parameter λ in the CDR version of $\mathcal{L}_{\text{eff}}$ is related to the renormalization of α_s.

5.2 Operators in FDH and DRED

In FDH and DRED, the basis of operators needs to contain additional terms involving ϵ-scalars. We use a basis constructed analogously to Eqs. (5.1) from gauge invariant operators and operators corresponding to field renormalization. Then there are two kinds of changes: there are modifications of the operators O_3 and O_4, and there are additional basis elements. The new basis operators correspond to the ϵ-scalar kinetic term, $\tilde{O}_1$, to the new parameters α_e and $\alpha_{4\epsilon}$, $\tilde{O}_3$ and $\tilde{O}_{4\epsilon,i}$, and to the field renormalization of $\tilde{A}^\mu$, $\tilde{O}_4$. The notation is chosen such that in all cases O_j and $\tilde{O}_j$ have a similar structure:

$$O_1 = -\frac{1}{4} \hat{F}_a^{\mu\nu} \hat{F}_{\mu\nu,a}, \tag{5.6a}$$

$$O_2 = 0, \tag{5.6b}$$

$$O_3 = \frac{i}{2} \overline{\psi} \overleftrightarrow{\hat{\not{D}}} \psi - g_e \overline{\psi} \tilde{\not{A}} \psi, \tag{5.6c}$$

$$O_4 = \hat{A}_a^\nu (\hat{D}^\mu \hat{F}_{\mu\nu})_a + g_s f_{abc} (\partial^\mu \tilde{A}_a^\nu) \hat{A}_{\mu,b} \tilde{A}_{\nu,c} - g_s \overline{\psi} \hat{\not{A}} \psi - (\partial^\mu \overline{c}_a)(\partial_\mu c_a), \tag{5.6d}$$

$$O_5 = (\hat{D}^\mu \partial_\mu \overline{c}_a) c_a, \tag{5.6e}$$

$$\tilde{O}_1 = -\frac{1}{2} (\hat{D}^\mu \tilde{A}^\nu)_a (\hat{D}_\mu \tilde{A}_\nu)_a, \tag{5.6f}$$

$$\tilde{O}_3 = g_e \overline{\psi} \tilde{\not{A}} \psi, \tag{5.6g}$$

$$\tilde{O}_4 = \tilde{A}_a^\nu (\hat{D}^\mu \hat{D}_\mu \tilde{A}_\nu)_a, \tag{5.6h}$$

$$\tilde{O}_{4\epsilon,i} = \mathcal{O}(\tilde{A}^4). \tag{5.6i}$$

Since we consider massless QCD there is no ϵ-scalar mass term. Like in Eq. (2.4), operators involving four ϵ-scalars are not needed explicitly.

This set of operators differs in a crucial way from the CDR case. The difference between operators $\tilde{O}_1$ and $\tilde{O}_4$ is related to the total derivative $\Box\tilde{A}^\mu\tilde{A}_\mu$. Hence, the basis for space-time integrated operators (zero-momentum insertions) does not coincide with the one for non-integrated operators (non-vanishing momentum insertions). As discussed by Spiridonov in Ref. [6], in such a case his method cannot be used. Therefore, in FDH and DRED it is not possible to derive complete results for the operator mixing analogous to Eqs. (5.4) and (5.5).

This implies two difficulties: First, the two-loop renormalization of $\tilde{O}_1$ and the corresponding parameter λ_ϵ cannot be obtained from a priori known two-loop QCD renormalization constants but need to be determined from an explicit two-loop off-shell calculation. Second, the off-shell Green functions get contributions from unphysical, gauge non-invariant operators, so the full operator mixing needs to be taken into account.

We have carried out the explicit one-loop calculations to obtain all required one-loop results for Z_{1j} and $Z_{\tilde{1}j}$. The results are

$$\delta Z_{11}^{1\mathrm{L}} = \left(\frac{\alpha_s}{4\pi}\right)\left[\left(-\frac{11}{3}+\frac{N_\epsilon}{6}\right)C_A + \frac{2}{3}N_F\right]\frac{1}{\epsilon}, \tag{5.7a}$$

$$\delta Z_{\tilde{1}1}^{1\mathrm{L}} = 0, \tag{5.7b}$$

$$\delta Z_{1\tilde{1}}^{1\mathrm{L}} = 0, \tag{5.7c}$$

$$\delta Z_{\tilde{1}\tilde{1}}^{1\mathrm{L}} = \left[\left(\frac{\alpha_s}{4\pi}\right)(-3)C_A + \left(\frac{\alpha_e}{4\pi}\right)N_F - \left(\frac{\alpha_{4\epsilon}}{4\pi}\right)(1-N_\epsilon)C_A\right]\frac{1}{\epsilon}, \tag{5.7d}$$

$$\delta Z_{13}^{1\mathrm{L}} = 0, \tag{5.7e}$$

$$\delta Z_{\tilde{1}3}^{1\mathrm{L}} = \left(\frac{\alpha_e}{4\pi}\right)\frac{N_\epsilon}{2}C_F\frac{1}{\epsilon}, \tag{5.7f}$$

$$\delta Z_{14}^{1\mathrm{L}} = \left(\frac{\alpha_s}{4\pi}\right)\frac{3}{4}C_A\frac{1}{\epsilon}, \tag{5.7g}$$

$$\delta Z_{\tilde{1}4}^{1\mathrm{L}} = 0, \tag{5.7h}$$

$$\delta Z_{1\tilde{4}}^{1\mathrm{L}} = \left(\frac{\alpha_s}{4\pi}\right)\left(-\frac{3}{2}\right)C_A\frac{1}{\epsilon}, \tag{5.7i}$$

$$\delta Z_{\tilde{1}\tilde{4}}^{1\mathrm{L}} = \left(\frac{\alpha_s}{4\pi}\right)\frac{1}{2}(3-\xi)C_A\frac{1}{\epsilon}, \tag{5.7j}$$

$$\delta Z_{15}^{1\mathrm{L}} = 0, \tag{5.7k}$$

$$\delta Z_{\tilde{1}5}^{1\mathrm{L}} = 0. \tag{5.7l}$$

Renormalization constants involving operators $\tilde{O}_3$ or $\tilde{O}_{4\epsilon,i}$ are not needed for the calculations in the present paper. The renormalization constants (5.7a)-(5.7d) agree with those given in Ref. [34]. The only gauge-dependent quantity is $Z_{\tilde{1}\tilde{4}}^{1\mathrm{L}}$. This is due to the fact that operator $\tilde{O}_4$ is related to the field renormalization of the ϵ-scalars. In all other renormal-

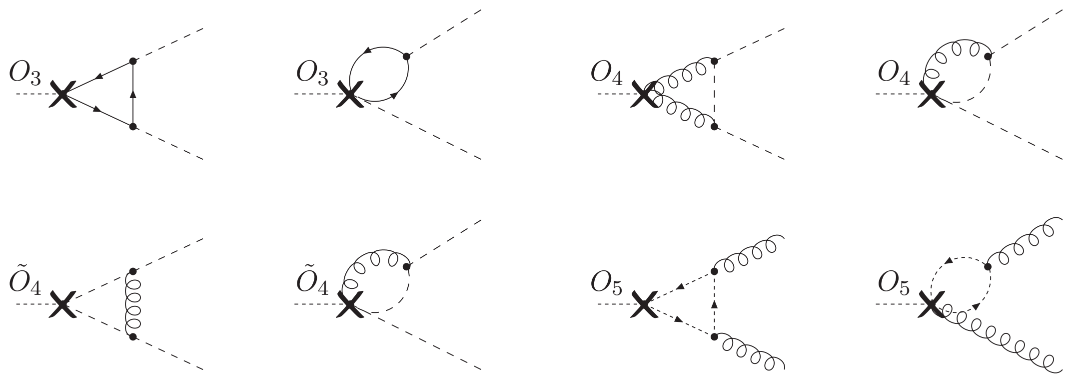

Figure 3. Sample one-loop counterterm diagrams originating from operators O_3, O_4, $\tilde{O}_4$ and O_5.

ization constants related to field renormalization the gauge-dependent parts incidentally cancel out.

With these results the bare effective Lagrangian can be written as

$$\mathcal{L}_{\rm eff}^{\rm bare} = H \sum_j \left(\lambda\, Z_{1j} O_{j,\rm bare} + \lambda_\epsilon\, Z_{\tilde{1}j} O_{j,\rm bare} \right), \tag{5.8}$$

where the sum runs over all operators in Eqs. (5.6). Sometimes it is useful to write this using multiplicative renormalization constants for λ and λ_ϵ as

$$\mathcal{L}_{\rm eff}^{\rm bare} = Z_\lambda \lambda H O_{1,\rm bare} + Z_{\lambda_\epsilon} \lambda_\epsilon H O_{\tilde{1},\rm bare} + \ldots , \tag{5.9}$$

suppressing operators not present at tree level, such that $\lambda Z_\lambda = \lambda Z_{11} + \lambda_\epsilon Z_{\tilde{1}1}$ and similar for Z_{λ_ϵ}.

The one-loop counterterm effective Lagrangian involving the renormalization constants of Eqs. (5.7) is then given by

$$\mathcal{L}_{\rm eff}^{\rm 1LCT} = H \sum_j \left(\lambda\, \delta Z^{\rm 1L}_{1j} O_j + \lambda_\epsilon\, \delta Z^{\rm 1L}_{\tilde{1}j} O_j \right). \tag{5.10}$$

We have now all ingredients for the one-loop counterterm diagrams $\Gamma^{\rm 1LCT,b}_{HA^\mu A^\nu}$ relevant for the computation of $H \to gg$, where the gluons are either D-dimensional gauge fields or ϵ-scalars. These counterterm contributions arise from one-loop counterterm diagrams with one insertion of $\mathcal{L}_{\rm eff}^{\rm 1LCT}$. Sample diagrams are given in Fig. 3. They show insertions of operators O_3, O_4, $\tilde{O}_4$ and O_5. Feynman rules for operator insertions are given in appendix A.2.

The calculation shows that all these operators generate non-vanishing contributions to $\Gamma^{\rm 1LCT,b}_{HA^\mu A^\nu}$. However, in the extraction of the form factors and two-loop renormalization constants to be discussed in the next section there are cancellations, and O_4 is the only new operator which contributes.

6 Two-loop renormalization constants of λ and λ_ϵ

Putting together the results from the previous three sections it is possible to calculate the two-loop renormalization constants $\delta Z_\lambda^{\rm 2L}$ and $\delta Z_{\lambda_\epsilon}^{\rm 2L}$ appearing in Eq. (5.9). They can be obtained from a complete off-shell two-loop calculation and the requirement that the corresponding Green-functions are UV finite after renormalization:

$$\Gamma^{\rm 2L}_{HA^\mu A^\nu} + \Gamma^{\rm 1LCT,a}_{HA^\mu A^\nu} + \Gamma^{\rm 2LCT,a}_{HA^\mu A^\nu} + \Gamma^{\rm 1LCT,b}_{HA^\mu A^\nu} + \Gamma^{\rm 2LCT,b}_{HA^\mu A^\nu}\Big|^{\rm off\text{-}shell}_{\rm UV\ div.} = 0. \tag{6.1}$$

All ingredients except the last term are computed in the previous sections, and Eq. (6.1) is then used to extract $\delta Z_\lambda^{\rm 2L}$ and $\delta Z_{\lambda_\epsilon}^{\rm 2L}$. The result for $\delta Z_\lambda^{\rm 2L}$ is:

$$\begin{aligned}\delta Z_\lambda^{\rm 2L} =& \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg\{ C_A^2 \left[\frac{\frac{121}{9} - \frac{11}{9}N_\epsilon + \frac{N_\epsilon^2}{36}}{\epsilon^2} + \frac{-\frac{34}{3} + \frac{7}{3}N_\epsilon}{\epsilon}\right] \\ &\qquad + C_A N_F \left[\frac{-\frac{44}{9} + \frac{2}{9}N_\epsilon}{\epsilon^2} + \frac{10}{3\epsilon}\right] + C_F N_F \frac{2}{\epsilon} + N_F^2 \frac{4}{9\epsilon^2}\Bigg\} \\ &+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right) C_F N_F \frac{\left(-1 - \frac{\lambda_\epsilon}{\lambda}\right) N_\epsilon}{2\epsilon}.\end{aligned} \tag{6.2}$$

Since the off-shell calculations have been done numerically with the help of FIESTA [44] the analytical expressions have been obtained by rounding to a least common denominator. The numerical uncertainty is less than $\frac{1}{72}$ for the terms of the order $\mathcal{O}(\epsilon^{-2})$ and $\frac{1}{6}$ for the terms of the order $\mathcal{O}(\epsilon^{-1})$.

Result (6.2) is not new; it agrees with Ref. [34], where it has been obtained using Spiridonov's method. The recalculation serves as a test of the setup and the results given in the previous sections. At the same time a comparison with Ref. [34] confirms that Eq. (6.2) is actually exactly correct, in spite of numerical uncertainties.

In the same way, we obtain the renormalization constant $\delta Z_{\lambda_\epsilon}^{\rm 2L}$:

$$\begin{aligned}\delta Z_{\lambda_\epsilon}^{\rm 2L} =& \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg\{ C_A^2 \left[\frac{\frac{49}{4} + \frac{5}{4}N_\epsilon}{\epsilon^2} + \frac{-\frac{113}{24} + \frac{71}{24}N_\epsilon + \frac{\lambda}{\lambda_\epsilon}\left(2 - \frac{N_\epsilon}{2}\right)}{\epsilon}\right] + C_A N_F \left[-\frac{1}{\epsilon^2} + \frac{\frac{5}{6} - 2\frac{\lambda}{\lambda_\epsilon}}{\epsilon}\right]\Bigg\} \\ &+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right)\Bigg\{ C_A N_F \left[-\frac{3}{\epsilon^2} + \frac{\frac{3}{2} + 3\frac{\lambda}{\lambda_\epsilon}}{\epsilon}\right] + C_F N_F \left[-\frac{3}{\epsilon^2} + \frac{\frac{5}{2} - 3\frac{\lambda}{\lambda_\epsilon}}{\epsilon}\right]\Bigg\} \\ &+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_{4\epsilon}}{4\pi}\right) C_A^2 \left(1 - N_\epsilon\right)\left[\frac{6}{\epsilon^2} + \frac{-4 - 3\frac{\lambda}{\lambda_\epsilon}}{\epsilon}\right] \\ &+ \left(\frac{\alpha_e}{4\pi}\right)^2 \Bigg\{ C_A N_F \frac{-\frac{3}{2} + \frac{3}{4}N_\epsilon}{\epsilon} + C_F N_F \left[-\frac{3N_\epsilon}{2\,\epsilon^2} + \frac{3 - \frac{7}{4}N_\epsilon}{\epsilon}\right] + \frac{N_F^2}{\epsilon^2}\Bigg\} \\ &+ \left(\frac{\alpha_e}{4\pi}\right)\left(\frac{\alpha_{4\epsilon}}{4\pi}\right) C_A N_F \left(1 - N_\epsilon\right)\left[-\frac{2}{\epsilon^2} + \frac{3}{2\epsilon}\right] \\ &+ \left(\frac{\alpha_{4\epsilon}}{4\pi}\right)^2 C_A^2 \left(1 - N_\epsilon\right)\left[\frac{-\frac{5}{4} - N_\epsilon}{\epsilon^2} + \frac{15}{8\epsilon}\right].\end{aligned} \tag{6.3}$$

Compared to Eq. (6.2) this result is more complicated and includes all combinations of the three couplings α_s, α_e and $\alpha_{4\epsilon}$. This result is new; as described in Sec. 5 it cannot be obtained using Spiridonov's method. The numerical uncertainty is less than $\frac{1}{48}$ for all terms. A forthcoming comparison with a prediction of the infrared structure of $H \to \tilde{g}\tilde{g}$ will confirm that expression (6.3) is exactly correct [48].

7 UV renormalized form factors of gluons and ϵ-scalars

Now that all renormalization constants are known it is possible to calculate the two-loop form factors of gluons and ϵ-scalars in the FDH and DRED scheme. We present the results in two ways: First, we give results with independent couplings needed to determine the IR anomalous dimensions of gluons and ϵ-scalars; second, we give simplified results, where all couplings are set equal. These can be viewed as the final results for the UV renormalized but IR regularized form factors. We give them including higher orders in the ϵ-expansion.

7.1 Results for independent couplings

The UV renormalized but IR divergent form factor for $H \to \hat{g}\hat{g}$ in DRED is given at the one-loop and two-loop level by

$$\begin{aligned}
&\bar{F}^{1\mathrm{L}}_{\hat{g}}(\alpha_s, \lambda_\epsilon/\lambda, N_\epsilon)\\
&=\Big(\frac{\alpha_s}{4\pi}\Big)\Bigg\{C_A\Bigg[-\frac{2}{\epsilon^2}+\frac{-\frac{11}{3}+\frac{N_\epsilon}{6}}{\epsilon}+\frac{\pi^2}{6}+\frac{\lambda_\epsilon}{\lambda}N_\epsilon+\epsilon\Big(-2+\frac{14}{3}\zeta(3)+3\frac{\lambda_\epsilon}{\lambda}N_\epsilon\Big)\Bigg]+\frac{2N_F}{3\epsilon}\Bigg\}\\
&\quad+\mathcal{O}(\epsilon^2), \qquad (7.1)
\end{aligned}$$

$$\begin{aligned}
&\bar{F}^{2\mathrm{L}}_{\hat{g}}(\alpha_s, \alpha_e, \lambda_\epsilon/\lambda, N_\epsilon)\\
&=\Big(\frac{\alpha_s}{4\pi}\Big)^2\Bigg\{C_A^2\Bigg[\frac{2}{\epsilon^4}+\frac{\frac{77}{6}-\frac{7}{12}N_\epsilon}{\epsilon^3}+\frac{\frac{175}{18}-\frac{\pi^2}{6}-N_\epsilon\Big(1+2\frac{\lambda_\epsilon}{\lambda}\Big)+\frac{N_\epsilon^2}{36}}{\epsilon^2}\\
&\qquad+\frac{-\frac{238}{27}-\frac{11}{36}\pi^2-\frac{25}{3}\zeta(3)+N_\epsilon\Big(\frac{49}{27}+\frac{\pi^2}{72}-\frac{29}{3}\frac{\lambda_\epsilon}{\lambda}\Big)+\frac{1}{6}\frac{\lambda_\epsilon}{\lambda}N_\epsilon^2}{\epsilon}\Bigg]\\
&\qquad+C_AN_F\Bigg[-\frac{7}{3\epsilon^3}+\frac{-\frac{13}{3}+\frac{2}{9}N_\epsilon}{\epsilon^2}+\frac{\frac{64}{27}+\frac{\pi^2}{18}+\frac{2}{3}\frac{\lambda_\epsilon}{\lambda}N_\epsilon}{\epsilon}\Bigg]+C_FN_F\frac{1}{\epsilon}+\frac{4N_F^2}{9\epsilon^2}\Bigg\}\\
&\quad-\Big(\frac{\alpha_s}{4\pi}\Big)\Big(\frac{\alpha_e}{4\pi}\Big)C_FN_F\frac{N_\epsilon}{2\epsilon}+\mathcal{O}(\epsilon^0). \qquad (7.2)
\end{aligned}$$

As mentioned in the beginning the $\hat{g}$ form factor in DRED is identical to the gluon form factor in FDH, and Eq. (7.2) agrees with the result given in Ref. [34].

Since there are no external ϵ-scalars in diagrams related to the gluon form factor internal ϵ-scalars have to be part of a closed ϵ-scalar loop or have to couple to a closed

fermion loop. Hence, the effective coupling λ_ϵ always appears together with at least one power of N_ϵ in Eqs. (7.1) and (7.2).

The ϵ-scalar form factor for $H \to \tilde{g}\tilde{g}$ in DRED is given by

$$
\begin{aligned}
&\bar{F}^{1\mathrm{L}}_{\tilde{g}}(\alpha_s, \alpha_e, \alpha_{4\epsilon}, \lambda/\lambda_\epsilon, N_\epsilon)\\
&=\Big(\frac{\alpha_s}{4\pi}\Big) C_A\bigg[-\frac{2}{\epsilon^2}-\frac{4}{\epsilon}-2+\frac{\pi^2}{6}+2\frac{\lambda}{\lambda_\epsilon}+\epsilon\Big(-4+\frac{\pi^2}{12}+\frac{14}{3}\zeta(3)+4\frac{\lambda}{\lambda_\epsilon}\Big)\bigg]\\
&\quad+\Big(\frac{\alpha_e}{4\pi}\Big)\frac{N_F}{\epsilon}+\Big(\frac{\alpha_{4\epsilon}}{4\pi}\Big) C_A\,(1-N_\epsilon)\bigg[2+\epsilon\Big(4-\frac{\pi^2}{12}\Big)\bigg]+\mathcal{O}(\epsilon^2),
\end{aligned}
\tag{7.3}
$$

$$
\begin{aligned}
&\bar{F}^{2\mathrm{L}}_{\tilde{g}}(\alpha_s, \alpha_e, \alpha_{4\epsilon}, \lambda/\lambda_\epsilon, N_\epsilon)\\
&=\Big(\frac{\alpha_s}{4\pi}\Big)^2\bigg\{C_A^2\bigg[\frac{2}{\epsilon^4}+\frac{\frac{27}{2}-\frac{N_\epsilon}{4}}{\epsilon^3}+\frac{\frac{281}{18}-\frac{\pi^2}{6}-\frac{N_\epsilon}{9}-4\frac{\lambda}{\lambda_\epsilon}}{\epsilon^2}\\
&\qquad\qquad+\frac{\frac{469}{216}-\frac{5}{12}\pi^2-\frac{25}{3}\zeta(3)+N_\epsilon\Big(\frac{233}{216}+\frac{\pi^2}{24}\Big)-16\frac{\lambda}{\lambda_\epsilon}}{\epsilon}\bigg]\\
&\qquad\quad+C_AN_F\bigg[-\frac{1}{\epsilon^3}-\frac{7}{9\epsilon^2}+\frac{\frac{113}{54}+\frac{\pi^2}{6}}{\epsilon}\bigg]\bigg\}\\
&\quad+\Big(\frac{\alpha_s}{4\pi}\Big)\Big(\frac{\alpha_e}{4\pi}\Big)\bigg\{C_AN_F\bigg[-\frac{2}{\epsilon^3}-\frac{4}{\epsilon^2}+\frac{-2-\frac{\pi^2}{6}+2\frac{\lambda}{\lambda_\epsilon}}{\epsilon}\bigg]+C_FN_F\bigg[-\frac{3}{\epsilon^2}+\frac{5}{2\epsilon}\bigg]\bigg\}\\
&\quad+\Big(\frac{\alpha_s}{4\pi}\Big)\Big(\frac{\alpha_{4\epsilon}}{4\pi}\Big)C_A^2\,(1-N_\epsilon)\bigg[-\frac{4}{\epsilon^2}+\frac{-16+\frac{\pi^2}{6}}{\epsilon}\bigg]\\
&\quad+\Big(\frac{\alpha_e}{4\pi}\Big)^2\bigg\{C_AN_F\bigg[\frac{-1+\frac{N_\epsilon}{2}}{\epsilon^2}+\frac{\frac{1}{2}-\frac{N_\epsilon}{4}}{\epsilon}\bigg]+C_FN_F\bigg[\frac{2-\frac{N_\epsilon}{2}}{\epsilon^2}+\frac{-1-\frac{N_\epsilon}{4}}{\epsilon}\bigg]+\frac{N_F^2}{\epsilon^2}\bigg\}\\
&\quad+\Big(\frac{\alpha_e}{4\pi}\Big)\Big(\frac{\alpha_{4\epsilon}}{4\pi}\Big)C_AN_F\,(1-N_\epsilon)\,\frac{2}{\epsilon}\\
&\quad+\Big(\frac{\alpha_{4\epsilon}}{4\pi}\Big)^2C_A^2(1-N_\epsilon)\frac{-3}{8\epsilon}+\mathcal{O}(\epsilon^0).
\end{aligned}
\tag{7.4}
$$

Compared to Eqs. (7.1) and (7.2) the result with external ϵ-scalars is more complicated and includes all combinations of the couplings α_s, α_e and $\alpha_{4\epsilon}$. In this result, like in all previous results, the evanescent coupling α_e appears always together with at least one power of N_F and the quartic coupling $\alpha_{4\epsilon}$ is always accompanied by a factor $(1 - N_\epsilon)$.

7.2 Results for equal couplings

During the renormalization process the couplings α_s, α_e, $\alpha_{4\epsilon}$ and λ, λ_ϵ have to be distinguished. After renormalization they can be set equal, giving a simpler form of the final

result.[4] The results for $N_\epsilon = 2\epsilon$ at the one(two)-loop level up to order $\mathcal{O}(\epsilon^4)$ ($\mathcal{O}(\epsilon^2)$) then read:

$$\begin{aligned}\bar{F}^{1\mathrm{L}}_{\hat{g}} = \Big(\frac{\alpha_s}{4\pi}\Big)\bigg\{ C_A\bigg[&-\frac{2}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{1}{3} + \frac{\pi^2}{6} + \epsilon\frac{14}{3}\zeta(3) + \epsilon^2\frac{47}{720}\pi^4 \\ &+ \epsilon^3\Big(\frac{62}{5}\zeta(5) - \frac{7}{18}\pi^2\zeta(3)\Big) + \epsilon^4\Big(\frac{949}{60480}\pi^6 - \frac{49}{9}\zeta(3)^2\Big)\bigg] \\ &+ \frac{2N_F}{3\epsilon}\bigg\} + \mathcal{O}(\epsilon^5),\end{aligned} \tag{7.5}$$

$$\begin{aligned}\bar{F}^{2\mathrm{L}}_{\hat{g}} = \Big(\frac{\alpha_s}{4\pi}\Big)^2\bigg\{ C_A^2\bigg[&\frac{2}{\epsilon^4} + \frac{77}{6\epsilon^3} + \frac{\frac{77}{9} - \frac{\pi^2}{6}}{\epsilon^2} + \frac{-\frac{400}{27} - \frac{11}{36}\pi^2 - \frac{25}{3}\zeta(3)}{\epsilon} \\ &+ \frac{5711}{162} + \frac{17}{9}\pi^2 - 33\zeta(3) - \frac{7}{60}\pi^4 \\ &+ \epsilon\Big(\frac{189767}{972} + \frac{65}{27}\pi^2 - \frac{1058}{27}\zeta(3) - \frac{1111}{2160}\pi^4 + \frac{71}{5}\zeta(5) + \frac{23}{18}\pi^2\zeta(3)\Big) \\ &+ \epsilon^2\Big(\frac{4972715}{5832} - \frac{233}{324}\pi^2 - \frac{26404}{81}\zeta(3) - \frac{307}{360}\pi^4 - \frac{341}{5}\zeta(5) \\ &\qquad + \frac{257}{1680}\pi^6 - \frac{11}{54}\pi^2\zeta(3) + \frac{901}{9}\zeta(3)^2\Big)\bigg] \\ + C_A N_F\bigg[&-\frac{7}{3\epsilon^3} - \frac{13}{3\epsilon^2} + \frac{\frac{76}{27} + \frac{\pi^2}{18}}{\epsilon} - \frac{916}{81} - \frac{5}{18}\pi^2 - 2\zeta(3) \\ &+ \epsilon\Big(-\frac{14603}{243} - \frac{8}{27}\pi^2 - \frac{604}{27}\zeta(3) - \frac{59}{1080}\pi^4\Big) \\ &+ \epsilon^2\Big(-\frac{366023}{1458} + \frac{127}{162}\pi^2 - \frac{4448}{81}\zeta(3) - \frac{257}{648}\pi^4 - \frac{98}{5}\zeta(5) + \frac{61}{27}\pi^2\zeta(3)\Big)\bigg] \\ + C_F N_F\bigg[&\frac{1}{\epsilon} - \frac{73}{6} + 8\zeta(3) + \epsilon\Big(-\frac{2045}{36} + \frac{7}{18}\pi^2 + \frac{92}{3}\zeta(3) + \frac{4}{27}\pi^4\Big) \\ &+ \epsilon^2\Big(-\frac{53269}{216} + \frac{263}{108}\pi^2 + \frac{1232}{9}\zeta(3) + \frac{46}{81}\pi^4 + 32\zeta(5) - \frac{20}{9}\pi^2\zeta(3)\Big)\bigg] \\ &+ \frac{4N_F^2}{9\epsilon^2}\bigg\} + \mathcal{O}(\epsilon^3),\end{aligned} \tag{7.6}$$

[4] If the results of Sec. 7.1 were not desired for independent couplings, the genuine two-loop diagrams could have been computed in a simpler way, with all couplings set equal from the beginning — this is what is done in many applications of FDH and DRED in the literature.

$$
\begin{aligned}
\bar{F}^{1\mathrm{L}}_{\tilde{g}} = \left(\frac{\alpha_s}{4\pi}\right)\Bigg\{ C_A\Bigg[& -\frac{2}{\epsilon^2} - \frac{4}{\epsilon} + 2 + \frac{\pi^2}{6} + \epsilon\frac{14}{3}\zeta(3) + \epsilon^2\frac{47}{720}\pi^4 \\
& + \epsilon^3\Big(\frac{62}{5}\zeta(5) - \frac{7}{18}\pi^2\zeta(3)\Big) + \epsilon^4\Big(\frac{949}{60480}\pi^6 - \frac{49}{9}\zeta(3)^2\Big)\Bigg] \\
& + \frac{N_F}{\epsilon}\Bigg\} + \mathcal{O}(\epsilon^5),
\end{aligned}
\tag{7.7}
$$

$$
\begin{aligned}
\bar{F}^{2\mathrm{L}}_{\tilde{g}} = \left(\frac{\alpha_s}{4\pi}\right)^2\Bigg\{ & C_A^2\Bigg[\frac{2}{\epsilon^4} + \frac{27}{2\epsilon^3} + \frac{\frac{64}{9} - \frac{\pi^2}{6}}{\epsilon^2} + \frac{-\frac{1211}{54} - \frac{\pi^2}{4} - \frac{25}{3}\zeta(3)}{\epsilon} \\
& + \frac{6052}{81} + \frac{263}{108}\pi^2 - \frac{323}{9}\zeta(3) - \frac{7}{60}\pi^4 \\
& + \epsilon\left(\frac{263363}{972} + \frac{1489}{324}\pi^2 - \frac{1655}{27}\zeta(3) - \frac{67}{120}\pi^4 + \frac{71}{5}\zeta(5) + \frac{23}{18}\pi^2\zeta(3)\right) \\
& + \epsilon^2\left(\frac{6457043}{5832} + \frac{6803}{972}\pi^2 - \frac{34459}{81}\zeta(3) - \frac{15221}{12960}\pi^4 - \frac{235}{3}\zeta(5)\right. \\
& \left. + \frac{257}{1680}\pi^6 - \frac{16}{27}\pi^2\zeta(3) + \frac{901}{9}\zeta(3)^2\right)\Bigg] \\
& + C_A N_F\Bigg[-\frac{3}{\epsilon^3} - \frac{52}{9\epsilon^2} + \frac{151}{27\epsilon} - \frac{1925}{162} - \frac{25}{54}\pi^2 - \frac{28}{9}\zeta(3) \\
& + \epsilon\left(-\frac{10538}{243} - \frac{46}{81}\pi^2 - \frac{922}{27}\zeta(3) - \frac{61}{720}\pi^4\right) \\
& + \epsilon^2\left(-\frac{291065}{1458} + \frac{419}{486}\pi^2 - \frac{8678}{81}\zeta(3) - \frac{3971}{6480}\pi^4\right. \\
& \left. - \frac{382}{15}\zeta(5) + \frac{203}{54}\pi^2\zeta(3)\right)\Bigg] \\
& + C_F N_F\Bigg[-\frac{1}{\epsilon^2} + \frac{1}{2\epsilon} - 41 - \frac{\pi^2}{3} + 12\zeta(3) \\
& + \epsilon\left(-\frac{669}{4} - \frac{3}{2}\pi^2 + \frac{196}{3}\zeta(3) + \frac{2}{9}\pi^4\right) \\
& + \epsilon^2\left(-\frac{4607}{8} - \frac{61}{12}\pi^2 + \frac{868}{3}\zeta(3) + \frac{67}{60}\pi^4 + 48\zeta(5) - \frac{10}{3}\pi^2\zeta(3)\right)\Bigg] \\
& + \frac{N_F^2}{\epsilon^2}\Bigg\} + \mathcal{O}(\epsilon^3).
\end{aligned}
\tag{7.8}
$$

8 Conclusions

We have computed the $H \to gg$ amplitudes at the two-loop level in the FDH and DRED scheme and presented the $\overline{\text{MS}}$ renormalized on-shell results up to the order ϵ^2. In DRED, this involves two different amplitudes for $H \to \hat{g}\hat{g}$ and $H \to \tilde{g}\tilde{g}$ with external gluons/ϵ-scalars. The computation is motivated because it contains key elements which constitute important building blocks for further computations, and because it is essential for the complete understanding of the infrared divergence structure of FDH and DRED amplitudes.

The renormalization procedure has been described in detail. It is less trivial than in many QCD calculations in CDR, since not only the strong coupling needs to be renormalized but also evanescent couplings of the ϵ-scalar. The computation provides a further example of the well-known fact that regardless of whether FDH or DRED is used, the evanescent couplings have to be renormalized independently.

Further, the renormalization of the effective dimension-5 operators involves mixing with new, ϵ-scalar dependent operators. A suitable basis of operators has been provided. One unavoidable fact is that the extended operator space contains operators which are total derivatives. As a result the required operator mixing renormalization constants cannot be obtained in the same elegant way of Ref. [6] as in CDR. Instead, they had to be obtained from explicit one- and two-loop off-shell calculations.

The results for the UV renormalized but infrared divergent form factors can also be used to complete the study of the general infrared divergence structure of two-loop amplitudes in FDH and DRED, begun in Ref. [33, 34]. From general principles it is known that all infrared divergences can be expressed in terms of cusp and parton anomalous dimensions. The results of the present paper allow to extract the final missing two-loop anomalous dimension for external ϵ-scalars. This extraction, together with further checks and results, will be presented in a forthcoming paper [48], where the infrared structure will also be investigated by a SCET approach.

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A Appendix

A.1 Projectors and form factors of gluons and ϵ-scalars

According to its Lorentz structure the on-shell Green-function $\Gamma^{\text{on-shell}}_{H\hat{A}^\mu\hat{A}^\nu}$ can be represented as

$$\Gamma^{\text{on-shell}}_{H\hat{A}^\mu\hat{A}^\nu} = a\,(p\cdot r)\,\hat{g}^{\mu\nu} + b\,p^\nu r^\mu + c\,p^\mu r^\nu + d\,p^\mu p^\nu + e\,r^\mu r^\nu, \tag{A.1}$$

where the coefficients $a \dots e$ are momentum-dependent quantities, and coefficient a is the gluon form factor. Due to QCD Ward-identities the relation $a = -b$ holds, see e. g. Ref. [1].

Accordingly, the on-shell Green-function $\Gamma^{\text{on-shell}}_{H\tilde{A}^\mu\tilde{A}^\nu}$ with external ϵ-scalars can be represented as

$$\Gamma^{\text{on-shell}}_{H\tilde{A}^\mu\tilde{A}^\nu} = f\,(p\cdot r)\,\tilde{g}^{\mu\nu}, \tag{A.2}$$

where we refer to f as ϵ-scalar form factor. All coefficients of the covariant decomposition can be extracted with appropriate projection operators that are given below.

In the off-shell case the UV divergence structure of $\Gamma_{H\hat{A}^\mu\hat{A}^\nu}$ can be represented in a more specific way as

$$\Gamma_{H\hat{A}^\mu\hat{A}^\nu}\Big|^{\text{off-shell}}_{\text{UV div.}} = \left[A + A'\,\frac{p^2+r^2}{(p\cdot r)}\right](p\cdot r)\,\hat{g}^{\mu\nu} + B\,p^\nu r^\mu + C\,p^\mu r^\nu + D\,p^\mu p^\nu + E\,r^\mu r^\nu, \tag{A.3}$$

where the coefficients $A\ldots E$ are now momentum-independent. Since these divergences can be absorbed by counterterms corresponding to operators O_1 and O_4 the relation $A=-B$ again holds, see e. g. Feynman rules (A.6) and (A.12). Due to this there are two possibilities of extracting coefficient A, which corresponds to the desired renormalization constant $\delta Z^{2\text{L}}_\lambda$: The first one is to extract the coefficient of $(p\cdot r)\,\hat{g}^{\mu\nu}$ and neglect terms $\propto p^2, r^2$; the second is to extract coefficient $-B$. We checked explicitly that the relations $a=-b$ and $A=-B$ hold throughout the paper.

Again, the covariant decomposition with external ϵ-scalars is much simpler and reads:

$$\Gamma_{H\tilde{A}^\mu\tilde{A}^\nu}\Big|^{\text{off-shell}}_{\text{UV div.}} = \left[F + F'\,\frac{p^2+r^2}{(p\cdot r)}\right](p\cdot r)\,\tilde{g}^{\mu\nu}. \tag{A.4}$$

The desired coefficient for the computation of $\delta Z^{2\text{L}}_{\lambda_\epsilon}$ is F. Accordingly, we extract the coefficient of $(p\cdot r)\,\tilde{g}^{\mu\nu}$ and neglect terms $\propto p^2, r^2$.

The corresponding projection operators are:

$$\begin{aligned} P^{\mu\nu}_{g,(p\cdot r)\hat{g}^{\mu\nu}} = \Big\{ &\hat{g}^{\mu\nu}\left[(p\cdot r)^2 - p^2r^2\right] \\ &- (p^\nu r^\mu + p^\mu r^\nu)(p\cdot r) \\ &+ p^\mu p^\nu r^2 + r^\mu r^\nu p^2 \Big\} \frac{1}{(D-2)(p\cdot r)\left[(p\cdot r)^2 - p^2r^2\right]}, \end{aligned} \tag{A.5a}$$

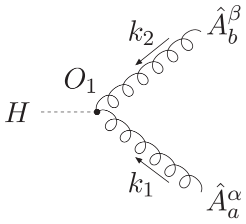
$$\begin{aligned} P^{\mu\nu}_{g,p^\nu r^\mu} = \Big\{ &\hat{g}^{\mu\nu}\,(p\cdot r)\left[p^2r^2 - (p\cdot r)^2\right] \\ &+ p^\nu r^\mu\left[(p\cdot r)^2 + p^2r^2(D-2)\right] + p^\mu r^\nu\,(p\cdot r)^2\,(D-1) \\ &+ (p^\mu p^\nu r^2 + r^\mu r^\nu p^2)(p\cdot r)(1-D) \Big\} \frac{1}{(D-2)\left[(p\cdot r)^2 - p^2r^2\right]^2}, \end{aligned} \tag{A.5b}$$

$$P^{\mu\nu}_{\tilde{g},(p\cdot r)\tilde{g}^{\mu\nu}} = \frac{\tilde{g}^{\mu\nu}}{N_\epsilon(p\cdot r)}. \tag{A.5c}$$

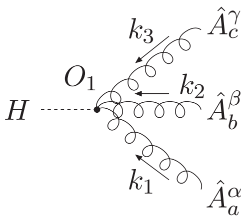
A.2 Feynman rules

In the following we give Feynman rules according to operators $O_1, \tilde{O}_1, O_4$ and $\tilde{O}_4$ that are needed for the renormalization in the FDH and DRED scheme. Feynman rules including four ϵ-scalars are not relevant in this paper and are not given explicitly.

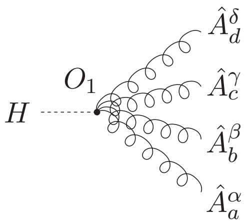
• Feynman rules according to the Lagrangian term $\lambda H O_1$:



$$= \quad i\lambda \left[(k_1 \cdot k_2)\, \hat{g}^{\,\alpha\beta} - k_1^{\,\beta}\, k_2^{\,\alpha} \right] \delta^{ab} \tag{A.6}$$

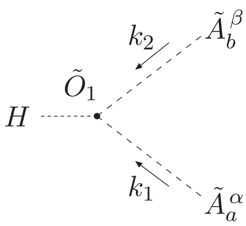


$$= \quad -\lambda\, g_s f^{abc} \times \begin{bmatrix} \hat{g}^{\alpha\beta}\, (k_1 - k_2)^\gamma \\ + \hat{g}^{\beta\gamma}\, (k_2 - k_3)^\alpha \\ + \hat{g}^{\gamma\alpha}\, (k_3 - k_1)^\beta \end{bmatrix} \tag{A.7}$$

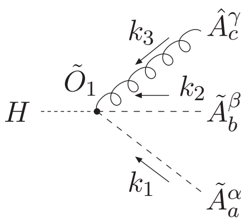


$$= \quad -i\lambda\, g_s^2 \times \begin{bmatrix} \hat{g}^{\alpha\beta} \hat{g}^{\gamma\delta} \left(f^{ace} f^{bde} + f^{ade} f^{bce} \right) \\ + \hat{g}^{\alpha\gamma} \hat{g}^{\beta\delta} \left(f^{abe} f^{cde} - f^{ade} f^{bce} \right) \\ - \hat{g}^{\alpha\delta} \hat{g}^{\beta\gamma} \left(f^{abe} f^{cde} + f^{ace} f^{bde} \right) \end{bmatrix} \tag{A.8}$$

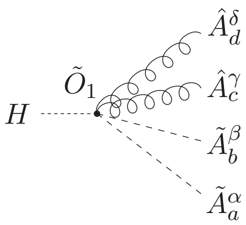
• Feynman rules according to the Lagrangian term $\lambda_\epsilon H \tilde{O}_1$



$$= \quad i\lambda_\epsilon \left[(k_1 \cdot k_2)\, \tilde{g}^{\,\alpha\beta} \right] \delta^{ab} \tag{A.9}$$

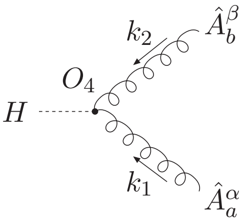


$$= \quad -\lambda_\epsilon\, g_s f^{abc}\, \tilde{g}^{\,\alpha\beta}\, (k_1 - k_2)^\gamma \tag{A.10}$$

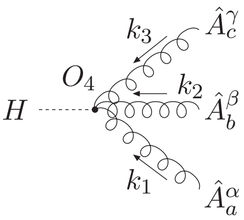


$$= \quad -i\lambda_\epsilon\, g_s^2\, \tilde{g}^{\,\alpha\beta} \hat{g}^{\gamma\delta} \left(f^{ace} f^{bde} + f^{ade} f^{bce} \right) \tag{A.11}$$

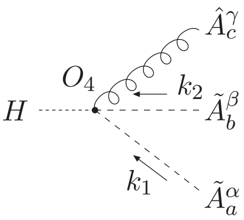
• Feynman rules according to the Lagrangian term HO_4:



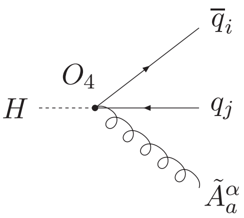
$$= \quad -i\left[\,\left(k_1^2+k_2^2\right)\hat{g}^{\,\alpha\beta}-\left(k_1^{\alpha}\,k_1^{\beta}+k_2^{\alpha}\,k_2^{\beta}\right)\right]\delta^{ab} \tag{A.12}$$



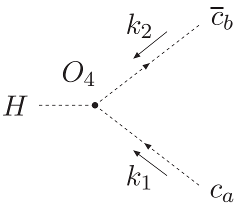
$$= \quad 3\,g_s f^{abc}\times\begin{bmatrix}\hat{g}^{\alpha\beta}\,(k_1-k_2)^{\gamma}\\ +\,\hat{g}^{\beta\gamma}\,(k_2-k_3)^{\alpha}\\ +\,\hat{g}^{\gamma\alpha}\,(k_3-k_1)^{\beta}\end{bmatrix} \tag{A.13}$$



$$= \quad -g_s f^{abc}\,\tilde{g}^{\,\alpha\beta}\,(k_1-k_2)^{\gamma} \tag{A.14}$$

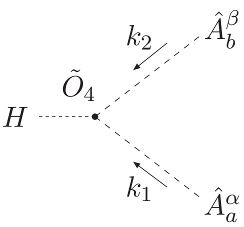


$$= \quad -ig_s\,\hat{\gamma}^{\alpha}\,(T^a)_{ij} \tag{A.15}$$

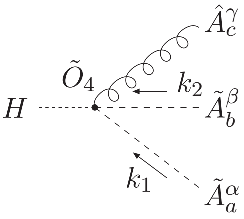


$$= \quad i\,(k_1\cdot k_2)\,\delta_{ab} \tag{A.16}$$

• Feynman rules according to the Lagrangian term $H\tilde{O}_4$:



$$= \quad -i\left(k_1^2+k_2^2\right)\tilde{g}^{\,\alpha\beta}\,\delta^{ab} \tag{A.17}$$



$$= \quad -2\, g_s f^{abc}\, \tilde{g}^{\alpha\beta}\, (k_1 - k_2)^\gamma \tag{A.18}$$

References

[1] R. V. Harlander, *Virtual corrections to $g\ g \to H$ to two loops in the heavy top limit*, *Phys.Lett.* **B492** (2000) 74–80, [hep-ph/0007289].

[2] S. Moch, J. Vermaseren, and A. Vogt, *Three-loop results for quark and gluon form-factors*, *Phys.Lett.* **B625** (2005) 245–252, [hep-ph/0508055].

[3] P. Baikov, K. Chetyrkin, A. Smirnov, V. Smirnov, and M. Steinhauser, *Quark and gluon form factors to three loops*, *Phys.Rev.Lett.* **102** (2009) 212002, [arXiv:0902.3519].

[4] T. Gehrmann, E. Glover, T. Huber, N. Ikizlerli, and C. Studerus, *Calculation of the quark and gluon form factors to three loops in QCD*, *JHEP* **1006** (2010) 094, [arXiv:1004.3653].

[5] T. Gehrmann, E. Glover, T. Huber, N. Ikizlerli, and C. Studerus, *The quark and gluon form factors to three loops in QCD through to O(eps^2)*, *JHEP* **1011** (2010) 102, [arXiv:1010.4478].

[6] V. Spiridonov, *Anomalous Dimension of $G^2_{\mu\nu}$ and β-function*, *CERN Document Server* (1984) IYaI–P–0378.

[7] H. Kluberg-Stern and J. Zuber, *Ward Identities and Some Clues to the Renormalization of Gauge Invariant Operators*, *Phys.Rev.* **D12** (1975) 467–481.

[8] S. D. Joglekar and B. W. Lee, *General Theory of Renormalization of Gauge Invariant Operators*, *Annals Phys.* **97** (1976) 160.

[9] A. Cherchiglia, M. Sampaio, and M. Nemes, *Systematic Implementation of Implicit Regularization for Multi-Loop Feynman Diagrams*, *Int.J.Mod.Phys.* **A26** (2011) 2591–2635, [arXiv:1008.1377].

[10] L. C. Ferreira, A. Cherchiglia, B. Hiller, M. Sampaio, and M. Nemes, *Momentum routing invariance in Feynman diagrams and quantum symmetry breakings*, *Phys.Rev.* **D86** (2012) 025016, [arXiv:1110.6186].

[11] R. Pittau, *A four-dimensional approach to quantum field theories*, *JHEP* **1211** (2012) 151, [arXiv:1208.5457].

[12] A. Cherchiglia, L. Cabral, M. Nemes, and M. Sampaio, *(Un)determined finite regularization dependent quantum corrections: the Higgs boson decay into two photons and the two photon scattering examples*, *Phys.Rev.* **D87** (2013) 065011, [arXiv:1210.6164].

[13] A. M. Donati and R. Pittau, *Gauge invariance at work in FDR: $H \to \gamma\gamma$*, *JHEP* **1304** (2013) 167, [arXiv:1302.5668].

[14] R. Pittau, *QCD corrections to $H \to gg$ in FDR*, *Eur.Phys.J.* **C74** (2014) 2686, [arXiv:1307.0705].

[15] W. Siegel, *Supersymmetric Dimensional Regularization via Dimensional Reduction*, *Phys.Lett.* **B84** (1979) 193.

[16] Z. Bern and D. A. Kosower, *The Computation of loop amplitudes in gauge theories*, *Nucl.Phys.* **B379** (1992) 451–561.

[17] Z. Kunszt, A. Signer, and Z. Trocsanyi, *One loop helicity amplitudes for all 2 → 2 processes in QCD and N=1 supersymmetric Yang-Mills theory*, *Nucl.Phys.* **B411** (1994) 397–442, [hep-ph/9305239].

[18] S. Catani, S. Dittmaier, and Z. Trocsanyi, *One loop singular behavior of QCD and SUSY QCD amplitudes with massive partons*, *Phys.Lett.* **B500** (2001) 149–160, [hep-ph/0011222].

[19] S. Catani, M. Seymour, and Z. Trocsanyi, *Regularization scheme independence and unitarity in QCD cross-sections*, *Phys.Rev.* **D55** (1997) 6819–6829, [hep-ph/9610553].

[20] I. Jack, D. Jones, and K. Roberts, *Dimensional reduction in nonsupersymmetric theories*, *Z.Phys.* **C62** (1994) 161–166, [hep-ph/9310301].

[21] I. Jack, D. Jones, and K. Roberts, *Equivalence of dimensional reduction and dimensional regularization*, *Z.Phys.* **C63** (1994) 151–160, [hep-ph/9401349].

[22] D. Stöckinger, *Regularization by dimensional reduction: consistency, quantum action principle, and supersymmetry*, *JHEP* **0503** (2005) 076, [hep-ph/0503129].

[23] A. Signer and D. Stöckinger, *Factorization and regularization by dimensional reduction*, *Phys.Lett.* **B626** (2005) 127–138, [hep-ph/0508203].

[24] A. Signer and D. Stöckinger, *Using Dimensional Reduction for Hadronic Collisions*, *Nucl.Phys.* **B808** (2009) 88–120, [arXiv:0807.4424].

[25] R. van Damme and G. 't Hooft, *Breakdown of Unitarity in the Dimensional Reduction Scheme*, *Phys.Lett.* **B150** (1985) 133.

[26] W. Siegel, *Inconsistency of Supersymmetric Dimensional Regularization*, *Phys.Lett.* **B94** (1980) 37.

[27] W. Beenakker, H. Kuijf, W. van Neerven, and J. Smith, *QCD Corrections to Heavy Quark Production in p anti-p Collisions*, *Phys.Rev.* **D40** (1989) 54–82.

[28] J. Smith and W. van Neerven, *The Difference between n-dimensional regularization and n-dimensional reduction in QCD*, *Eur.Phys.J.* **C40** (2005) 199–203, [hep-ph/0411357].

[29] R. Harlander, P. Kant, L. Mihaila, and M. Steinhauser, *Dimensional Reduction applied to QCD at three loops*, *JHEP* **0609** (2006) 053, [hep-ph/0607240].

[30] R. Harlander, D. Jones, P. Kant, L. Mihaila, and M. Steinhauser, *Four-loop beta function and mass anomalous dimension in dimensional reduction*, *JHEP* **0612** (2006) 024, [hep-ph/0610206].

[31] P. Kant, R. Harlander, L. Mihaila, and M. Steinhauser, *Light MSSM Higgs boson mass to three-loop accuracy*, *JHEP* **1008** (2010) 104, [arXiv:1005.5709].

[32] W. B. Kilgore, *Regularization Schemes and Higher Order Corrections*, *Phys.Rev.* **D83** (2011) 114005, [arXiv:1102.5353].

[33] W. B. Kilgore, *The Four Dimensional Helicity Scheme Beyond One Loop*, *Phys.Rev.* **D86** (2012) 014019, [arXiv:1205.4015].

[34] C. Gnendiger, A. Signer, and D. Stöckinger, *The infrared structure of QCD amplitudes and $H \to gg$ in FDH and DRED*, *Phys.Lett.* **B733** (2014) 296–304, [arXiv:1404.2171].

[35] T. Becher and M. Neubert, *On the Structure of Infrared Singularities of Gauge-Theory Amplitudes*, *JHEP* **0906** (2009) 081, [arXiv:0903.1126].

[36] T. Becher and M. Neubert, *Infrared singularities of scattering amplitudes in perturbative QCD*, *Phys.Rev.Lett.* **102** (2009) 162001, [arXiv:0901.0722].

[37] E. Gardi and L. Magnea, *Infrared singularities in QCD amplitudes*, *Nuovo Cim.* **C32N5-6** (2009) 137–157, [arXiv:0908.3273].

[38] L. Magnea, V. Del Duca, C. Duhr, E. Gardi, and C. D. White, *Infrared singularities in the high-energy limit*, *PoS* **LL2012** (2012) 008, [arXiv:1210.6786].

[39] C. Anastasiou, S. Beerli, and A. Daleo, *The Two-loop QCD amplitude $gg \to h,H$ in the Minimal Supersymmetric Standard Model*, *Phys.Rev.Lett.* **100** (2008) 241806, [arXiv:0803.3065].

[40] T. Hahn, *Generating Feynman diagrams and amplitudes with FeynArts 3*, *Comput.Phys.Commun.* **140** (2001) 418–431, [hep-ph/0012260].

[41] M. Jamin and M. E. Lautenbacher, *TRACER: Version 1.1: A Mathematica package for gamma algebra in arbitrary dimensions*, *Comput.Phys.Commun.* **74** (1993) 265–288.

[42] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations*, *Int.J.Mod.Phys.* **A15** (2000) 5087–5159, [hep-ph/0102033].

[43] A. Smirnov, *Algorithm FIRE – Feynman Integral REduction*, *JHEP* **0810** (2008) 107, [arXiv:0807.3243].

[44] A. Smirnov and M. Tentyukov, *Feynman Integral Evaluation by a Sector decomposiTion Approach (FIESTA)*, *Comput.Phys.Commun.* **180** (2009) 735–746, [arXiv:0807.4129].

[45] M.-x. Luo, H.-w. Wang, and Y. Xiao, *Two loop renormalization group equations in general gauge field theories*, *Phys.Rev.* **D67** (2003) 065019, [hep-ph/0211440].

[46] S. Larin and J. Vermaseren, *The Three loop QCD Beta function and anomalous dimensions*, *Phys.Lett.* **B303** (1993) 334–336, [hep-ph/9302208].

[47] W. Deans and J. A. Dixon, *Theory of Gauge Invariant Operators: Their Renormalization and S Matrix Elements*, *Phys.Rev.* **D18** (1978) 1113–1126.

[48] A. Broggio, C. Gnendiger, A. Signer, D. Stöckinger, and A. Visconti, *SCET approach to the regularization scheme dependence of QCD amplitudes*, *to be published* (2015).